

The Lonely Runner Conjecture

December 30, 2021

Beyond its open and pre-eminent stature in the field of mathematics, the lonely runner conjecture, at least to us runners, remains something of a home, a frustration to endure. Its last true advance was made a half-century ago by a group of Mexicans led by number theorist and 1996 Fields medalist Dr Niño-Mora who, by way of reducing the problem to a certain acyclic graph, proved theoretically, that is, without resorting to experimentation, the conjecture's truth in the case of seven runners. Though we runners are rarely afforded information from without, we are each morning at the break of dawn notified through those high-mounted speakers surrounding our track (towards which we so seldom raise our eyes) that, notwithstanding the Herculean essays by the most formidable mathematicians of our generation, the conjecture in the case of eight runners remains impenetrable. Accordingly, the continued commitment of us runners (five men, including myself, and three women) to this most important of human experiments is, as ever, appreciated.

You wanted to know, fair one, in the first place, how it was my and my fellow runners' speeds were decided on. Is that correct? What I can say, is that I am quite able to run faster, much faster, in fact, than the speed at which they have me. By this, I do not mean to suggest they are compassionate ones, that they have decided upon our speeds so as not to overwork us, like driving horses. Far be it from me, a mere runner, to guess at the intentions of the men behind the speakers. Indeed, it is not easy-running for all of us runners, unfortunately. Old Josef, for example---Ah, there he is now, coming around the bend, bound to pass us soon enough---old Josef was not assigned a speed apposite to his advanced age and audibly

creaking body. They have him running faster than me. But he never says a thing about it, mind you. Always he sports the most pleasant of countenances, issuing only the wisest of words. To say he has my respect, that everyday I aim to adopt a sameness to his virtuous deportment would be an understatement so severe as to encroach into the realm of lies. He, among all my fellow runners, I consider blessed. That they have assigned for poor Josef a speed just out of his reach is nothing short of a crime. Certainly, he will run on, committed to the conjecture as we all are, but we runners all know, whether we voice it or not, that at his speed ordained, Josef has at most a week, maybe two, of running left in him. His fate, like the fate of Georg will be the same. It is shameful to admit, but, in the main, I hardly think about him. I have my own running on which to concentrate, you see, as we all do if we are to maintain our speeds, evince good-will. It is not easy, I tell you. I am quite certain the others feel bad, worse than bad for old Josef, but such is our fate. We do as our abilities allow.

I was asked the other day by one of my fellow runners, out of the blue, in one of our little interviews as we playfully like to call our brief conversations, my opinion on why we had been chosen. "Of all people---" she said, "the sad, the enraged, unsung---why us?" The question struck me as odd. Not because it was silly; on the contrary, I thought it very fine. What I found odd, rather, was that so basic a question had never come up in our little interviews. Why had we been chosen, or, more narrowly (for I never have been comfortable answering questions on behalf of others; scarcely am I comfortable answering on behalf of myself) why had I been chosen? Looking objectively at myself, from above myself, so to speak, there was to my mind nothing very special about me. I never was too quick in body or brain; my memory, on a level with young children, can hardly extend beyond the calendar year, let alone to a time before my running. I would not be my first choice, as a runner, in a word. A thing I do have in spades, though, now that I am on it, is my affinity for mindless repetition. And by that I mean the more banal, the more inane, the more mindless and repetitive a task, the easier and more comforting I find its doing; its staying power, near infinite. There are men I know, my father being one (it is a wonder I did not inherit his tendency), given not to repetition but to progression

as the supreme means by which living is made tolerable. But I have never been so very charmed, as far as I can recollect, by tasks with definitive beginnings and middles and ends, tasks from which at any point during its doing one has only to cast up their eyes to gauge with unequivocal precision the progress made, the percentage remaining, which, of course, pushes such men onwards all the more, to complete their good work in the knowledge they have just now surpassed, say, the 5% mark. Already one-twentieth of the task complete! What joy! Such has never appealed to me. I was never a member of that clan, have always had a bent towards tasks of a quieter self-same sort.

Had she asked me instead: Why do I persist? Why do I continue in this way? Why do not I simply walk away, quit, halt this endless life of running? Had she asked me these instead, I would have said, well, perhaps the aforesaid. Always there is a cloud of suchlike questions looming over my head all day and night, ever darkly swelling, its manifold susurrations all sounding off at once. With no paper on which to write them, I bear the question all in my head, holding on to what I can. A shopper circumambulating a grocer's store without a basket, piling into her arms item after item, will eventually reach that point whereat items begin to tumble away onto the floor, eggs breaking, cruel emotions---anxiety, indignation, despair---mounting one other, self-assembling, until she simply has no desire to hold aught; nevertheless, she convinces herself to push on, you see, try more, not because above others God loves a tryer, but because she must return to her culling, time after time, regardless of how absurd, to go on trying, holding more than her talents allow. So, I have no basket in which to hold my questions, they float unfettered in that amorphous cloud; no sooner do new ones develop than old ones drop away, never able to accumulate. Said dually---as above, so below---you could say I am a child in a sandpit digging then refilling the same old hole again and again in the hopes of finding himself at the bottom of his natal land.

This is not for you. I am sorry, fair one. I am not used to others so interested in my thoughts, which are in a jumble, I know. Perhaps you can return to them, my thoughts, one day, with a gentler eye, and unknot what you can, truss them with string. But to return to your very reasonable question, Why do not I simply decamp from this eternal track, this unending running? Yes. A very reasonable

question; not silly at all.

I have been running for so long, lifting and planting the soles of my feet at my assigned speed for so long, it hardly feels a demand on my person at all, certainly no more than, say, the beating of my heart or the blooming of my lungs, the skipping of my thoughts. Running about this track, while it may appear unbearable to you, is, after all, to me, one of the truly glorious means by which to ease the passing of time. Its soothing regularity, you see, partakes of the first regularity: the thrum of the womb. We, after having been expelled into the electric day, our nerves wretchedly exposed and put on fire, ever seek for the remainder of our days, even into adulthood, to re-create the pulsing womb, its rhythms, its comforts. Is it such an oddity, then, that we runners persist as we do? Besides, let me remind you what is at stake: if confirmed, the lonely runner conjecture could burgeon an entirely new body of mathematics into which the efforts of mathematician the world over would pour for the next half-century. What does it matter, said differently, the feelings, desires of one man, a mere runner, when compared to the greatest of mathematical conjectures?

(Speaking now of the conjecture, as a brief aside, before I forget, one small detail that has bothered me now and again, though not always, but at times, is the method by which the conjecture is being tested. Now, I am no mathematician, but from what I can understand, the conjecture asserts that for any distinct speeds assigned to its runners, eventually one will become lonely. That our eventual loneliness must hold regardless our assigned speeds seems significant. For does it not stand to reason the speeds at which we currently run are merely a single instance of the problem parameters, one of an uncountable number of settings under which the conjecture may be verified? Must not, in other words, we eight runners run at every possible speed if the conjecture proper is to be confirmed or falsified? If so, owing to there being an infinitude of speeds (at this stage I am fully aware I have come to the limits of my rather limited mind) must not this experiment be configured and reconfigured in an innumerable number of ways, one for each unique combination of speeds, thereby rendering this experiment, in effect, indomitable? And while on the subject, what of the lonely runner conjecture with more than eight runners? In its original formulation the conjecture was posed for any number

of runners, not just eight. Even if, therefore, the men behind the speakers could establish the conjecture in the case of eight runners, which, by the aforementioned reasoning appears unlikely, what of the still infinity of cases in which the number of runners exceeds eight? Clearly, I have no answer to these questions. I do not even have a clear sentiment toward my unclarity. Such are merely a sample of the questions that constitute my ever looming cloud, the one I told you of. Questions that emerge, then fall away.)

Sorry, what was that? Do I get scared sometimes? Yes. Some nights, after the sun has set and the hot humidity of the day turns to that cold clamminess of the sick, and the full moon, swarmed by orbiting clouds that seem to want to cling to it like lost angels, takes on an unhealthy yellow and appears too large, too close to the earth---on some nights, like those, when I am passed by another runner, Josef, say, who by day is simply old Josef, the best man I know, but by night appears a grotesque hobbling thing from whom, owing to our fixed speeds, I can do nothing to escape, I wish, at those moments, to scream. Of course, I never do. If you were to film me at such times it would be a waste, a photograph would suffice, for you would capture a countenance of stone, no more changing than the tempo at which my feet tread over this asphalt. An odd anticipatory fear, I know. The moment he passes I want to jump out of my skin, my eyes become those of a terrified horse, all whites with pupils shivering to the sides, wanting to but also not wanting to look at what approaches from behind. Worse still is that barbaric knee-jerk response that follows, to bring violence to those who scare us. At such moments, when my mind is running wild, it is not my fellow runner, my Josef, who passes, but that which bottoms most fears: death itself; I want to haul him right off the track, stomp him bloody, to death. Barbaric, I know. (But also, do not you think, fair one, a curious conceit? To bring death to death itself. A recursive absurdity of sorts. Like lifting yourself out of a wood by your own hair, or, more recursive still, like blenching in the sudden knowledge that the trees among which you walk are in fact your own head-hairs standing on ends and no sooner do you pinch something small out of the top of your head than a giant hand (your own hand, you recognize) descends through the trees above you, bending their boughs, catching and lifting you by the hair off the forest floor, your own scalp.

That recursive. Perhaps the urge to kill death in the night is an expression of a latent desire of mine to free myself from this interminable situation. I do not know. It is hard to describe. All true feelings, I suppose, in their infinite complexity lie beyond the reach of language, its fumbling metaphors. There is nothing new in what I have said.) But I should say, not to leave a trace of ambiguity on your mind, that there are no attacks, not even verbal altercations. In fact, at night, when Josef passes, perhaps sensing my consternation, perhaps knowing suchlike feelings when others pass him, he respectfully quiets during those five minutes we exchange positions. We do not, as is our wont under the sun, greet one another with smiles, picking up our little interview whence we last left it, comfortably falling into the old rhythms of old friends, but, rather, we remain quiet as he passes, as if there was nothing but the night, not even ourselves.

Ah, let us admit a short interlude, fair one---good Josef is approaching! Wave, smile if you would like. You are too small for him to notice.

"Hey! Good Josef!" (Always this way I greet my friend.)

"Franz, how goes it?" (And he me thus.)

"Beautiful day, Josef!" (I took this line from him; usually he says it. I must be nervous.)

"Yes indeed, another beautiful day."

"We have been lucky."

"How are your shins, Franz?"

"Still young, thank you. And yours, still troublesome?"

"At times. Though I may've found a technique to overcome them. Perhaps I'll tell you about it next time round. Don't worry yourself. We'll see."

"We'll see." (With pleasantries over, we begin now our little interview proper.)

"So, Franz."

"Josef!"

"I passed Josephine on the way here."

"Josephine!"

"Yes, dear girl."

"Certainly."

"As beautiful as ever. She brings to my mind a lovely blue body of water."

"The weather has been beautiful, Josef."

"Water, Franz."

"Josef."

"Josephine isn't doing so well it seems."

"What did she say?"

"Nothing, really. Just a feeling."

"Feelings are not mere things, Josef. They are the unconscious threatening to trespass into the conscious, as Jung said. Well, not in so many words."

"I sure had a feeling after my little interview with her. I think she is planning to leave the track."

"She said this."

"No, she didn't."

"You felt she wanted to say this."

"No, I didn't."

"You feel this to be true."

"Yes."

"No."

"Yes."

"Then she said this, Josef, without saying this."

"I being a fast runner, and she a slow runner, we hardly have time to speak, Franz. You know this. Perhaps it was her smile, so full of sorrow I wanted to stop right then and hold her, that said it. We ran in the beautiful weather and she commented benignly on it, looking the entire time beyond the track to the bluey mountains, regarding them as if tracking a slow-moving floater inside her eye, guiding her over the landscape like the shadow of a butterfly. Then there was a small shift in her gaze and a tightening of her lips and fists, then I saw her look past them, the floater, the mountains, through them even, and her eyes locked onto something. I have never seen anything like it. She was not lost in some reverie, believe you me, I have been lost to enough of those to know the difference; she could actually see something beyond the mountains. And I tell you, Franz, seeing her that way, smiling, it made me sadder than I've ever been in all my life. Sad like waking from a dream that brings strangely close, across the vast distances of time, your mother who is calling to you some ways down the shore to come and see about a seashell she had found in purple sunset, the colour you enjoy, smiling and point to it with her finger, her long silver hair shimmering

under a broad-brimmed hat while seagulls call and struggle against the wind high in the sky and the surf makes the lapping sounds of the bath water in which she washed you as a child, and you wake all at once and realize that you were never there, nor she, and that she is dead and you'll never see her again. That sad."

"Until next time, Josef!"

"Franz."

Such is the way our little interview move from start to end. In truth, fair one, I have not a clue what Josef was going on about, only that it was about Josephine.

When it comes to Josephine, I do not so much know if I am in love with her or it is simply that, as is the wont of all men past a certain age, my yearning for companionship has become general. She is beautiful, certainly, and the only runner with whom I could imagine myself running away. I suppose I would if ever she asked. And if you are wondering, Yes, whenever I pass her, that other beastly urge (distinct from the desire to kill death; though on the opposite side of the same coin, you could say) swells inside me. I am no longer me at such times, she, no longer my Josephine. Watching her lithe and vixen figure prancing away from me, her pony tail swishing back and forth in perfect metronomic andante, her likewise pendular hips knocking side to side, snapping like a fist at the moment of greatest amplitude, I see this and am filled darkly by something orthogonal to companionship. She exists to be victimized. I, to catch and abuse her. (I am certain, by symmetry, Josef feels likewise toward me as he passes.) I can no more identify with such atavistic desires than a person can with the plethora of microorganisms that lie caged in their digestive tract. I have, though, come to accept such things are a part of me, come from inside me, my forebears. I would ignore them if I could. But always she looks back, my Josephine. Firstly, at my approaching shadow, then, directly at me, meeting my gaze with her singularly lovely green eyes, redolent of my motherland, Her grassy hills and holly yews, and those beastly urges vanish like a nightmare upon waking. She, my Josephine, must think me an idiot or near-idiot, for I never say more than a grunt and, whenever asked about myself, issue only a foolish half-conscious smile. Only after passing her do I think back on the fool I have made of myself, that I have yet again wasted the chance to speak with her, tell her how I love her,

how I wish to abdicate from my post, to slow down and match her speed that we may together away ourselves from this wretched track, her by my side, she understanding me, me her.

(It will be hours before I pass her again. I suppose with a supernatural effort I could share myself with Josephine, my appetite for love and resistance to pain, or feelings of tenderness, of voluptuous pleasures, of vaguely blurred regrets. But to share myself is not my main, such bravery is, I know, improbable in the extreme. I have, by the way, already forgotten what I and Josef spoke about.)

Yet, I have perhaps been inaccurate in speaking, calling this a "wretched track". Someone, I cannot remember who or how far back it was, once asked me: "Running as you do, over the same circular track day in and out, each of you near-identical copies of yourselves, gazing across the circumscribed field hardly able to distinguish whether it is you approaching or the one being approached, do you runners not ever swap places?" At the time I did not know so much what he was getting at; in fact, I had the distinct feeling he was asking in coded language a secret question about himself. In any event, he clearly had no interest in what I had to say. Perhaps he had already found his answer and merely wanted it said in a different voice. What I said to him frankly, perhaps too boldly was: "In truth, Sir, I no longer see myself as a self. I can hardly say I exist at all. I observe, perhaps, as the mountain goat observes, never knowing if he is in the midst of his goathood, or how much left of it there is to enjoy. He quietly, simply regards, in a condition of profound abstraction, the world, as colourful thoughts alight and drink from the placid lakes of his mind then flit away, scarcely leaving a ripple. So, I. I run and thoughts stream right through me, colouring faintly, with a sort of melancholy hue, the time of day, the pitch of the earth as measured by the sun's declining rays, the coolness of the breeze, the pain in my heart." He began, after I answered him thus, muttering something to qualify what he had asked, as though to justify himself. I, after a comprehensive gaze, saw where things were headed and quickly replaced what I had said with: "Swap places? Yes, I believe so." This seemed to allay whatever vexations he was suffering from. Over his face spread a warm satisfaction as if he had just consumed a delicious meal. I ought to have been outraged at him using me so, but said nothing. And as we ran together

at a common speed regarding the purpling, departing day, breathing in union, I did feel connected.

That was not you, fair one.

Ah, I see. You are not even here anymore. I have pushed you away, again. Speaking to myself again, carrying on my own conversation, out my mouth into my ears, both sayer and listener, as usual. Why not? Let me, then, fantasize about Josephine, for the time being, our out of doors wedding. I wonder if this time upon that vast moorland hard-swept by a wild Newfoundland wind with the vaguely layered mountains beyond and the fuzzy music issuing from the old amplifier, if from among the smiling crowd towards which Josephine, held arm in arm by her father, her first love, walks, her lovely face stiff with make-up, her liquid eyes bravely shimmering like a harbour full of masts and pennants, her mind under the azure of the sky both immense and round perhaps swooning with that unreal feeling of a reeling dream as her father says in an audible whisper, "Perfect," and she knows at that moment that that unspecified gravity which has heretofore kept her life from untethering has always been this moment wherein she may at last find peace, I wonder if upon that hill, from among the crowd, she will smile at me first.

I am still here; still here I am.

The trick, of course, has always been to push on, to continue even after having spent myself. It is not too unlike the lover who, after having arrived, goes on pleasuring his love, perhaps to a point of disgust. Indeed, there is something noble, necessary even, venturing beyond the beyond, reaching then surpassing that height you hitherto believed the absolute limit. Certainly, it is only natural to want to stop after mounting the first summit and declare it the principal peak, then relax. A young boy will, after receiving his first kiss, that pure and innocent exchange rendering forever his body blushed, transfigured, oft declare not a moment later that he could die. He has, to his mind, attained the apogee of human bliss and joy, that any feeling henceforth can at best be a pale echo of that first and only real thing. I do not know if there is another mountain on the mountain, I certainly do not continue my running in the hopes I will get there, for I know it was never meant for us runners, but I do know I shall keep running, that to do so is something, a thing that can be followed through; I move and I run and my heart goes on beating,

like the seawind who never stops combing of the forelocks of laughing children generation after generation, for there is no place elsewhere he seeks, no place in the silent heights of memory he pines for, no place like that place where he was if he could only keep going all along because it is not there; my thoughts, I realize, are becoming frenetic, dithering out of bounds, fraying; but I do not so much mind that they fray, for their fraying is the true music of our existence, I believe, our thoughts and counter-thoughts being the warp and weft of that ever fraying fabric subject to its own kind of entropy that reveals to us, thread by thread, the material of which we are made; it is what I was made for, after all, to endure, not to suffer, but to endure, like a machine in a bedroom, to continue on mechanically, like an old man with a failing memory asking from time to time for news of the son he has lost, to be confronted by notions of the abyss, comforted by the motions and the movements of my legs, the pumping of my arms, the feeling of the passing air, the sounds of the birds perched high upon the speakers, and I will forget all this, then remember it all, confusing it all with something that I had read long ago or recently dreamed, forgetting and remembering, as my heart like water over a spinning wheel breaks forever, and my mind fades away like a discoloured painting that can no longer be restored, succumbing to savage dreams of despair and of love and confusion and ever loneliness, which may seem like all too much, like drowning in a symphony of waves and wails of laughter mixing and spinning and spewing in purgatorial delight is all too much, but to endure it anyway, is a way, to endure.

Perhaps I have said this before; perhaps not.

And so, anyway, we eight runners (five men, including myself, and three women) continue running on our track, each of us, at our own steady speed, in our own distinct way ever asking, "At some point, continuing as we do, will one of us (it does not much matter who) eventually find themselves isolated, separate from the rest, lonely?"